

3. Is the MVT hypothesis satisfied? No

$f(x) = x^{\frac{1}{3}}$  on  $[-1, 1]$

if  $f$  cont on  $[a, b]$  yes  
diff on  $(a, b)$  no

$f'(x) = \frac{1}{3}x^{-2/3} = \frac{1}{3\sqrt[3]{x^2}}$  undefined if  $x=0$

Oct 10-9:29 AM

#### 4.3 Connecting $f'$ and $f''$ with the graph of $f$

##### first derivative test for local extrema of continuous functions

$y' - 0 +$   $y$  has a min  $\checkmark$   
 $- * +$

$y$  has a min if  $y'$  changes from  $-$  to  $+$

$y' + 0 -$   $y$  has a max  $\checkmark$   
 $+ * -$

$y$  has a max if  $y'$  changes from  $+$  to  $-$

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find the local extrema:

$$y = (x^2 - 3)e^x$$

$$y' = 2xe^x + e^x(x^2 - 3) = 0$$

$$y' = e^x(2x + x^2 - 3) = e^x(x^2 + 2x - 3) = 0$$

$$y' = e^x(x+3)(x-1) = 0 \quad x = -3, 1$$

sign graph  $y' \begin{array}{c} + \quad - \quad + \\ -3 \quad 1 \end{array}$

max at  $x = -3$  because  $y'$  changes from  $+$  to  $-$   
max is  $y = 6e^{-3}$

min at  $x = 1$  because  $y'$  changes from  $-$  to  $+$   
min is  $y = -2e$

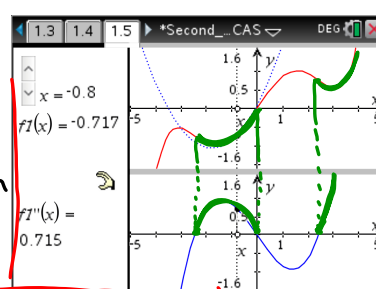
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concavity test

$y$  concave up  
 $y'' > 0$

$y$  concave down  
 $y'' < 0$

$y$  inflection point when  $y''$  changes sign



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Find all points of inflection for the graph of  $y = e^{-x^2}$ 

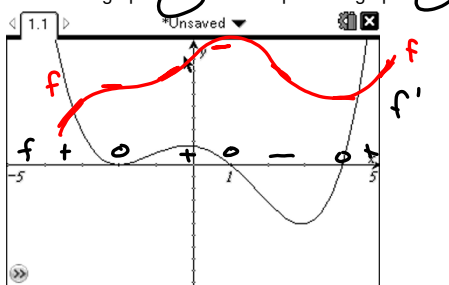
$$y' = (-2x)e^{-x^2}$$

$$y'' = (-2x)(-2x)e^{-x^2} + e^{-x^2}(-2) = 0$$

$$y'' = e^{-x^2}(4x^2 - 2) = 0 \quad x = \pm\sqrt{\frac{1}{2}}$$

$$y'' \begin{array}{c} + \quad - \quad + \\ -\sqrt{\frac{1}{2}} \quad \sqrt{\frac{1}{2}} \end{array}$$

infl. pts at  
 $x = \pm\sqrt{\frac{1}{2}}$   
 $y = e^{-1/2}$

This is the graph of  $f'$ . Sketch a possible graph of  $f$ 

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Find the local extrema using the second derivative test  
 $y = x^3 - 12x - 5$

Given the graph of  $f'$  sketch a possible graph of  $f$

